

# Wave-Based Teleoperation with Prediction

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## Abstract

Wave variables based on passivity and scattering theory provide a good tool for establishing bilateral teleoperation in the presence of a constant time delay. Recently these techniques have been extended to be used for a varying time delay, as in the case of Internet based teleoperation. Although stability is guaranteed for virtually any time delay, performance rapidly degrades for larger delays. In this paper a predictor derived from a modified Smith predictor along with a Kalman estimator and an energy regulator is used to enhance the performance of a wave-based teleoperator in the presence of a constant delay. Also the current wave transformation equations are extended to a more general case (better suited for systems with multiple degrees of freedom).

## 1 Introduction

In 1989 Anderson and Spong [1] proposed a control law for teleoperators with time delay based on passivity and scattering theory [3]. Although this method results in a controller which is stable for any non-varying time delay the authors made extensive use of network theory, modeling manipulators and manipulator control systems with  $n$ -dimensional (electrical) networks. For example, a single degree of freedom bilateral teleoperator would be represented as a network of resistors, inductors and capacitors. Representing a mechanical system with its electrical analogy is something that is not to the liking of a dynamicist or a mechanical engineer, as it tends to be non-intuitive in nature. Furthermore this approach is greatly complicated for systems with multiple degrees of freedom (see [2]).

Starting in the early 1990's Niemeyer and Slotine ([5], [6], [7] and [8]) proposed a more intuitive, physically motivated, passivity-based formalism to provide energy conservation and stability guarantees in the presence of time delay. Their idea too had its roots stemming from passivity and scattering theory, however the use of electrical networks to represent mechanical systems was eliminated, making their version of the same idea as that proposed by Anderson and Spong friendlier to

mechanicians. Recently researchers have extended the use of wave variables to variable delay, as in the case of the internet (see [9]).

One of the limitations of wave variables until now has been the degradation of performance as the delay increases. Although stability is guaranteed for virtually any time delay, the amount of information in transit at any given instant also increases with increasing delay. This results in what is known as wave-reflections, which is the rebounding of information between the master and slave across the communication line and consequently results in very large settling times for the teleoperator (see [6]). In this paper a predictor is proposed which estimates a correction (to enhance performance) and applies this value to the returning wave signal through an energy regulator (to ensure passivity). It is emphasized that the method proposed in this paper is valid only for a constant time delay.

Section 1 gives a brief background of the recent work in this area. Section 2 summarizes how wave variables work (for the unfamiliar reader). In section 3 new, more general wave transformations (better suited for systems with multiple degrees of freedom) are derived. Section 4 deals with the development of a wave-based predictor. Section 5 introduces an energy regulator, which ensures that the predictor does not inject too much energy into the system leading to instability. Section 6 shows some simulation results and finally some concluding remarks are given in section 7.

## 2 Background

For the purpose of illustration consider a single degree of freedom bilateral teleoperator. In this arrangement the master manipulator is a single degree of freedom crank mechanism bilaterally coupled to a similar (slave) mechanism via a communication link. Writing out the equation of motion for the master manipulator yields

$$J_m \ddot{\theta}_m + b_m \dot{\theta}_m = \tau_m \quad (1)$$

where  $J_m$  is the crank inertia,  $b_m$  is the crank damping,  $\ddot{\theta}_m$  is the crank acceleration,  $\dot{\theta}_m$  is the crank velocity and  $\tau_m$  the applied torque. The subscript  $m$  denotes

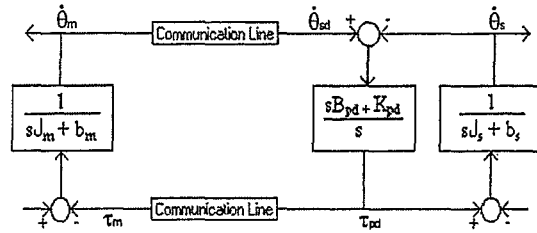


Figure 1: A single degree of freedom bilateral teleoperator.

variables representing the master manipulator. Similarly the equation of motion for the slave manipulator can be written as

$$J_s \ddot{\theta}_s + b_s \dot{\theta}_s = \tau_s \quad (2)$$

where the subscript  $s$  denotes the slave manipulator. In order for one side to track the other a PD controller is derived such that

$$\tau_m = -\tau_{pd} \quad (3)$$

$$\tau_s = \tau_{pd} \quad (4)$$

and

$$\tau_{pd} = K_{pd}(\theta_m - \theta_s) + B_{pd}(\dot{\theta}_m - \dot{\theta}_s) \quad (5)$$

where  $\tau_{pd}$  is the torque generated by the PD controller,  $K_{pd}$  is the proportional gain and  $B_{pd}$  the derivative gain. Figure 1 shows a block diagram of this arrangement. Notice the velocity vector propagates from the master to the slave, while the commanded force vector propagates from the slave to the master through the communication line. For as long as there is no delay, the system performs well. However as a small amount of delay is introduced in the communication line, performance starts to degrade and the system very quickly becomes unstable. It is apparent from the closed loop transfer function that with increasing time delay the closed loop system poles migrate into the right half plane. Anderson and Spong [1] attributed this problem solely to the non-passive nature of the communication link. In this arrangement the input-output variables across the communication line are related as

$$\begin{aligned} \dot{\theta}_{sd}(t) &= \dot{\theta}_m(t - T) \\ \tau_m(t) &= \tau_{pd}(t - T) \end{aligned} \quad (6)$$

Writing this in an input-output vector form yields

$$\begin{bmatrix} T_m \\ s\Theta_{sd} \end{bmatrix} = G_s(s) \begin{bmatrix} s\Theta_m \\ -T_{pd} \end{bmatrix} \quad (7)$$

The input on the right side is inverted as compared to the output on the left side. This is so that the inner product of the input and output vectors computes the total power-input to the communication lines

$$P_{in} = \dot{\theta}_m^T \tau_m - \dot{\theta}_{sd}^T \tau_{pd} \quad (8)$$

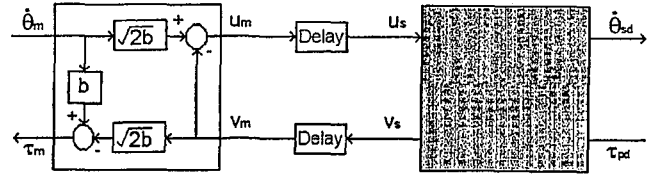


Figure 2: Wave based communication, by transforming velocity-force variables to wave variables before transmission and then back to velocity-force variable after transmission. Note: Elements for the right box are (symmetrically) identical to the left box.

It is known from linear systems theory that a system is passive if and only if the norm of the scattering operator

$$S(s) = (G_s(s) - I)(I + G_s(s))^{-1} \quad (9)$$

defined as

$$\|S\| = \sup_{\omega} \bar{\sigma}^{1/2}(S^*(j\omega)S(j\omega)) \quad (10)$$

is less than 1. However it can be shown that here this is not the case. Another way of looking at this is through equation 8, where it is seen that torque and velocity have a multiplicative dependence on the instantaneous power. This dependence can be eliminated using the wave transformations shown below

$$\begin{aligned} u_m(t) &= \frac{b\dot{\theta}_m(t) + \tau_m(t)}{\sqrt{2b}} \\ v_s(t) &= \frac{b\dot{\theta}_{sd}(t) - \tau_{pd}(t)}{\sqrt{2b}} \end{aligned} \quad (11)$$

where wave variables ( $u$  and  $v$ ), rather than power variables ( $\tau$  and  $\theta$ ), are transmitted across the communication line. The transmission process is now expressed as

$$\begin{aligned} u_s(t) &= u_m(t - T) \\ v_m(t) &= v_s(t - T) \end{aligned} \quad (12)$$

Figure 2 depicts how power variables are transformed to wave variables before transmission. Again writing this in an input-output vector form

$$\begin{bmatrix} T_m \\ s\Theta_{sd} \end{bmatrix} = G_w(s) \begin{bmatrix} s\Theta_m \\ -T_{pd} \end{bmatrix} \quad (13)$$

it can be shown that the norm of the scattering operator

$$S(s) = (G_w(s) - I)(I + G_w(s))^{-1} \quad (14)$$

is now equal to 1. In order to better understand how transforming velocity and force information into wave variables before transmission guarantees stability consider the left half of figure 2 (which represents a wave

transformation). From the work of Niemeyer [6], using wave variables the power-flow is now given as

$$P = \dot{\theta}_m^T \tau_m = \frac{1}{2} u_m^T u_m - \frac{1}{2} v_m^T v_m \quad (15)$$

Passivity requires that a system is passive if it cannot produce energy. Expressed mathematically

$$\int_0^t P d\tau = \int_0^t \dot{\theta}_m^T \tau_m d\tau = E_{store}(t) - E_{store}(0) + \int_0^t P_{diss} d\tau \quad (16)$$

where  $P_{diss}$  is the power dissipated,  $E(0)$  is the initial energy stored and  $E(t)$  is the energy stored at time  $t$ . Substituting equation (15) into (16) leads to

$$\int_0^t \frac{1}{2} v_m^T v_m d\tau = \int_0^t \frac{1}{2} u_m^T u_m d\tau - E_{store}(t) + E_{store}(0) - \int_0^t P_{diss} d\tau \quad (17)$$

which simplifies to

$$\int_0^t \frac{1}{2} v_m^T v_m d\tau \leq \int_0^t \frac{1}{2} u_m^T u_m d\tau + E_{store}(0) \quad (18)$$

Hence the system is passive if the energy in the outgoing wave is greater than or equal to the energy in the returning wave. Should the returning wave get delayed, then energy is only temporarily stored in the communication system and released later, still satisfying the passivity condition. Hence the need to eliminate the multiplicative dependence of what flows through the communication link on power-flow becomes obvious.

Although wave variables guarantee stability for virtually any constant delay, performance (mainly the settling time) rapidly decays with increasing delay. The goal of this study is to enhance the performance of a wave-based teleoperator through prediction techniques.

### 3 New Wave Transformation

Before the subject of prediction is dealt with we shall expand the wave transformation relation given by equation 11 to a more general case where the transformation parameters are matrices instead of a scalar. Writing the wave transformation relation of equation 11 in a more general form yields

$$\begin{aligned} u_m(t) &= A_w \dot{\theta}_m(t) + B_w \tau_m(t) \\ v_s(t) &= C_w \dot{\theta}_{sd}(t) - D_w \tau_{pd}(t) \end{aligned} \quad (19)$$

Similarly the expressions for  $v_m$  and  $u_s$  are given by

$$\begin{aligned} v_m(t) &= C_w \dot{\theta}_m(t) - D_w \tau_m(t) \\ u_s(t) &= A_w \dot{\theta}_{sd}(t) + B_w \tau_{pd}(t) \end{aligned} \quad (20)$$

where  $A_w$ ,  $B_w$ ,  $C_w$  and  $D_w$  are  $n \times n$  scaling matrices (and  $n$  is the number of degrees of freedom of the

teleoperator). For passivity we shall require that the relationship between power-flow into each side of the communication link and wave variables be the same as that given by equation 15, in other words

$$\dot{\theta}_m^T \tau_m = \frac{1}{2} u_m^T u_m - \frac{1}{2} v_m^T v_m \quad (21)$$

for the master side and

$$\dot{\theta}_s^T \tau_{pd} = -\frac{1}{2} u_s^T u_s + \frac{1}{2} v_s^T v_s \quad (22)$$

for the slave side. The independent variable  $t$  has been dropped for notational simplicity. Writing an input-output relationship across the communication link yields an expression similar to equation 13 where

$$G_w(s) = \begin{bmatrix} \frac{(1-e^{-2sT})}{1+e^{-2sT}} B_w^{-1} A_w & \frac{-2e^{-sT}}{1+e^{-2sT}} I \\ \frac{2e^{-sT}}{1+e^{-2sT}} I & \frac{(1-e^{-2sT})}{1+e^{-2sT}} A_w^{-1} B_w \end{bmatrix} \quad (23)$$

where  $I$  is an  $n \times n$  identity matrix. It can be shown that in order to satisfy equations 21 & 22 and that for the norm (equation 10) of the scattering operator (equation 14) to be equal to 1, the scaling matrices must satisfy<sup>1</sup> the following relations

$$\begin{aligned} A_w &= C_w \\ B_w &= D_w \\ I &= 2A_w B_w \end{aligned} \quad (24)$$

where matrices  $A_w$  and  $B_w$  are both invertable and symmetric. Note that if matrix  $A_w$  is set to  $\frac{b}{\sqrt{2b}}$ , equation 19 yields the original wave transformations (given by equation 11).

## 4 Wave Predictor

Figure 3 shows a possible arrangement of a predictor placed inside the wave junction. In this figure  $G_M(s)$  is the master manipulator's transfer function,  $G_S(s)$  is the transfer function of the slave and controller combined and  $G_P(s)$  is the transfer function of the predictor. The two rectangular boxes represent the wave transformations and  $G_R(s)$  is the total transfer function of the right side (i.e. the slave and the wave transformation combined).  $T_R$  and  $T_L$  represent the right and left time delays respectively. The box marked as *REGULATOR* shall be explained in the next section, for now the reader can assume that it is a summing junction.

### 4.1 Condition for zero residual error

Notice the predictor operates in the wave domain. If the predictor is not designed with care position tracking can

<sup>1</sup>The reader is referred to the author's Ph.D. thesis [4] for a more detailed proof of this analysis.

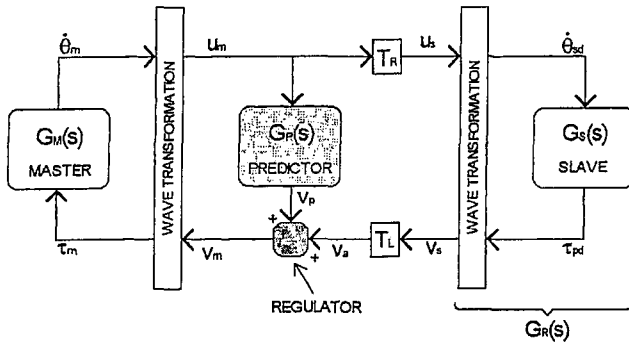


Figure 3: A possible arrangement of a predictor incorporated inside the wave junction.

not be guaranteed even though the velocity error might rapidly decay to zero. It can be shown that position information is encoded in the integral of the wave signals  $u$  and  $v$ , hence these integrals must be conserved for a zero steady state error. Writing the expression for position difference across the two wave junctions yields

$$\Delta\theta(t) = \theta_m(t) - \theta_{sd}(t) \quad (25)$$

which can be written in terms of wave variables as

$$\Delta\theta(t) = \frac{1}{2} A_w^{-1} \int_0^t (u_m + v_m - u_s - v_s) d\tau \quad (26)$$

Taking the Laplace transform of this and expanding, yields

$$\Delta\Theta(s) = \frac{1}{2s} A_w^{-1} (1 - e^{-sT_R} + G_P(s)) U_m(s) - \frac{1}{2s} A_w^{-1} (1 - e^{sT_L}) V_s(s) \quad (27)$$

At steady state when the transients have died out (and there is no more forced input at the master) the wave signals decay to zero, at which time we require that the position difference also be zero. In other words, given

$$\lim_{t \rightarrow \infty} u_m(t) = 0, \quad \lim_{t \rightarrow \infty} v_s(t) = 0 \quad (28)$$

we want

$$\lim_{t \rightarrow \infty} \Delta\theta(t) = \lim_{s \rightarrow 0} s\Delta\Theta(s) = 0 \quad (29)$$

Given that the limit of the wave signals exists, we can deduce from the final value theorem that

$$\lim_{t \rightarrow \infty} u_m(t) = 0 \quad \Rightarrow \quad \lim_{s \rightarrow 0} sU_m(s) = 0 \quad (30)$$

and

$$\lim_{t \rightarrow \infty} v_m(t) = 0 \quad \Rightarrow \quad \lim_{s \rightarrow 0} sV_s(s) = 0 \quad (31)$$

Hence it is required that for there to be no residual error

$$\lim_{s \rightarrow 0} G_P(s) = 0 \quad (32)$$

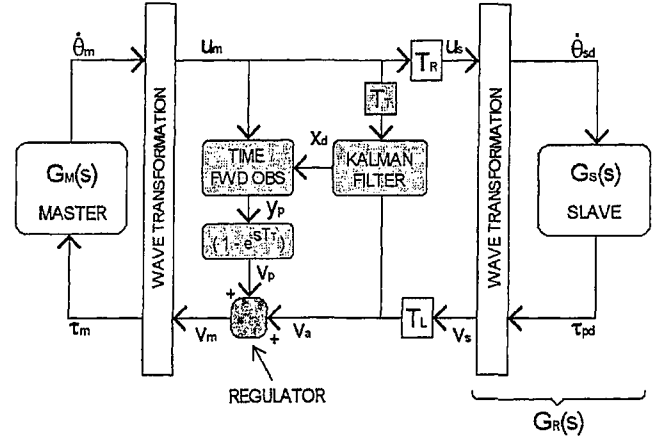


Figure 4: A possible arrangement of a predictor incorporated inside the wave junction.

## 4.2 Condition for passivity

In order to preserve passivity the predictor must not increase the total return energy of the system. Meaning that

$$\int_0^t \frac{1}{2} v_a^T v_a d\tau \geq \int_0^t \frac{1}{2} v_m^T v_m d\tau \quad (33)$$

should be met for all times. This condition can be explicitly enforced at the summing junction by a specially designed filter, which we shall refer to as the *REGULATOR*.

## 4.3 Predictor arrangement

One possible choice of the predictor could be

$$G_P(s) = (1 - e^{s(T_R + T_L)}) \hat{G}_R(s) \quad (34)$$

where  $\hat{G}_R(s)$  is a model of the entire right hand plant. Assuming that the regulator is a summing junction, the delayed wave signal is exactly cancelled, thus eliminating the transcendental term from the closed loop dynamics. This type of a predictor is also known as a Smith predictor (see [10] and [11]).

At first this type of a setup may look attractive for it is easy to implement and it satisfies the condition given by equation 32. However this method has a serious drawback. Suppose the initial conditions of the plant  $G_R(s)$  do not match that of the model  $\hat{G}_R(s)$ . In this case the prediction will not be effective and could possibly further deteriorate the performance. Even if the initial conditions match there could be a drift between the internal state of the model and that of the plant over time due to the plant interacting with the environment. In order to remedy this problem the predictor is modified to that shown in figure 4. In this arrangement a Kalman filter first estimates the internal state of the plant, which

is delayed by an amount  $T_T$ , where

$$T_T = T_R + T_L \quad (35)$$

Following this a time forward observer uses the output of the Kalman filter to march the state  $T_T$  seconds into the future and computes the output  $y_p$ , which is then used to obtain a prediction value  $v_p$  according to

$$V_p(s) = (1 - e^{sT_T})Y(s) \quad (36)$$

Notice that this expression satisfies the tracking condition given by equation 32.

#### 4.4 Predictor implementation

Starting with

$$\begin{aligned} \dot{x}(t) &= Ax(t) + Bu_m(t - T_R) \\ v_s(t) &= Cx(t) + Du_m(t - T_R) \end{aligned} \quad (37)$$

as the state space representation of the entire right hand side of the system (marked  $G_R(s)$  in figure 4) and defining a new state variable  $x_d(t) = x(t - T_L)$ , the above expression can be written as

$$\begin{aligned} \dot{x}_d(t) &= Ax_d(t) + Bu_m(t - T_T) \\ v_a(t) &= v_s(t - T_L) = Cx_d(t) + Du_m(t - T_T) \end{aligned} \quad (38)$$

Hence looking at equation 38 the entire right-hand side plant can be viewed as if it was driven by a control signal delayed  $T_T$  units of time and has an internal state  $x_d(t)$ . The Kalman filter<sup>2</sup> is used to estimate the internal state vector  $x_d(t)$  from the delayed input  $u_m(t - T_T)$  and the measured output  $v_a(t)$ .

The time forward observer now generates a predicted state vector  $x_p(t)$  (corresponding to the current input  $u_m(t)$ ), from the delayed state vector  $x_d(t)$  (which corresponds to the delayed input  $u_m(t - T_T)$ ) according to

$$x_p(t) = e^{AT_T}x_d(t) + \int_{t-T_T}^t e^{A(t-\tau)}Bu_m(\tau) d\tau \quad (39)$$

and finally the new output is computed as

$$y_p(t) = Cx_p(t) + Du_m(t) \quad (40)$$

For implementation ease the integral term in equation 39 can be computed according to the following state space model

$$\begin{aligned} \dot{z}(t) &= Az(t) + Bu_m(t) \\ g(t) &= z(t) - e^{AT_T}z(t - T_T) \end{aligned} \quad (41)$$

where it can be shown that

$$g(t) = \int_{t-T_T}^t e^{A(t-\tau)}Bu_m(\tau) d\tau \quad (42)$$

<sup>2</sup>A Luenberger observer could be used as an alternative.

Notice that the predictor does not require any initial conditions and that the Kalman filter will eventually converge to the correct internal state of the slave as viewed on the left side of the communication link. Because the internal state of the slave is directly effected when the slave interacts with the environment and the predictor relies on the Kalman filter to estimate the internal state of the slave, no measurements of the forces exerted by the remote environment onto the slave are needed.

## 5 Regulation

In order to guarantee passivity the condition depicted by equation 33 needs to be met. In other words the predictor must not increase the total energy contained in the returning wave  $v_m$ . This condition can be explicitly enforced through the use of a filter, which we refer to as a regulator. First we define  $v_t(t)$  as the sum of the returning wave  $v_a(t)$  and the prediction  $v_p(t)$ .

$$v_t(t) = v_a(t) + v_p(t) \quad (43)$$

The goal is to minimize the 'distance-to-go' defined as

$$D_{tg}(t) = \int_0^t (v_t(\tau) - v_m(\tau)) d\tau \quad (44)$$

For this purpose we define an energy reservoir

$$E_r(t) = \int_0^t (v_a^T(\tau)v_a(\tau) - v_m^T(\tau)v_m(\tau)) d\tau \quad (45)$$

which keeps track of the energy extracted by the regulator. The control law which computes  $v_m$  in order to drive  $D_{tg}(t)$  to zero based on the energy contained in the reservoir is then given by

$$v_m(t) = \alpha (1 - e^{-\beta E_r(t)}) D_{tg}(t) \quad (46)$$

where  $\alpha$  and  $\beta$  are both positive constant tuning parameters. At startup it would take a little time for the reservoir to build up, the size of which is governed by  $\beta$  while  $\alpha$  determines how fast  $D_{tg}(t)$  decays. Choosing  $\alpha$  and  $\beta$  to be both positive ensures that the energy reservoir (equation 45) is kept positive, hence the output  $v_m(t)$  and the distance  $D_{tg}(t)$  are always of the same sign.

## 6 Nonlinearity of the Regulator

Note that the dynamics of the regulator are indeed nonlinear, while it was assumed in deriving the zero-error condition given by equation (32) that what ever lies between the wave junctions is linear. However it should be noted that in this special case as the system approaches

steady state the prediction signal  $v_p$  has a net effect of zero. This can be shown as follows. Let its integral be given by

$$I_p(t) = \int_0^t v_p(t) dt = \int_0^t y_p(t) - y_p(t - T_T) dt \quad (47)$$

At steady state when the transients have decayed and there is no forced input at either side

$$y_p(t) \rightarrow 0 \Rightarrow I_p(t) \rightarrow 0 \quad (48)$$

Given that the integral of the prediction signal decays to zero at steady state, the integral of the returning wave  $v_a$  is unaffected. The fact that position information is encoded in the integral of wave signal (see equations 26 and 27) and that the final value of this integral with and without prediction is the same ensures that there is no steady state error between the master and the slave.

## 7 Simulation Results

The performance of the controller is illustrated in the simulation results which follow. The slave manipulator is comprised of a 3 degree of freedom parallel link, industrial robot built by Hyundai. The master manipulator is a 5 times scaled down version of the slave, hence is kinematically similar. The non-linear dynamics of both the master and slave manipulators are feedback linearized so that the resulting system is linear.

In the simulation the user moves the tip of the master manipulator along a desired trajectory for a period of 10 seconds. For the purpose of simulation of the forces applied by the user, the human arm is modeled as a simple mass-spring-damper system.

$$F_h(s) = Z_{arm}(s)(X_d(s) - X(s)) \quad (49)$$

where  $x_d$  is the desired position,  $x$  is the actual position and the arm impedance is given by

$$Z_{arm}(s) = Ms^2 + Bs + K \quad (50)$$

Finally the tip force applied by the user is mapped to the joints through the jacobian transformation. Figure 5 shows the tip position of the master and slave manipulators under conventional wave based teleoperation (i.e. without prediction). The desired trajectory commanded by the user is also shown. The forward and return time delays are set to 0.5 second, making the total round trip delay exactly 1 second. Because the slave is 5 times larger than the master, the desired tip position (commanded by the user's mind) and master's tip position profiles are scaled up by a factor of 5 so that they can be shown on the same plot for comparison. In this simulation the user moves the tip of the robot starting from the center of the plot, migrating towards the right along

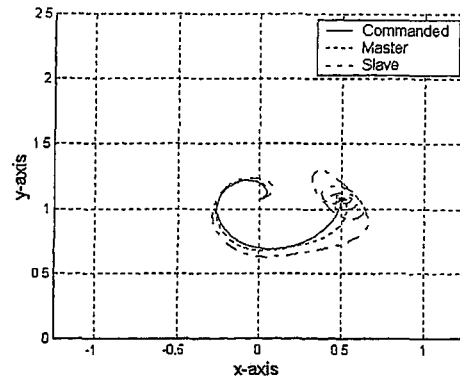


Figure 5: X-Y position of the tip with no prediction.

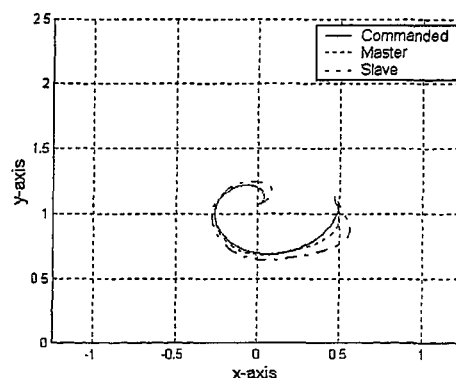


Figure 6: X-Y position of the tip with prediction.

a circular arc for a period of 10 seconds. The slave initially follows the master but eventually circles about the target position before coming to rest. Figure 6 shows the same system simulated with prediction. Notice this time the system does not circle the target as compared to figure 5. Figures 7 and 8 show plots for the joint angles, joint velocities and joint torque's for the same two simulation runs. Also plots for the difference in the joint angles and velocities between the master and slave are shown for both cases. Notice that when there is no prediction the system takes almost an additional 20 seconds to come to rest after the user stops moving the master manipulator (at the 10 second mark). However with prediction the system comes to rest with in 3 seconds after the user stops moving the master manipulator. A clear improvement!

## 8 Conclusion

Unlike advanced Smith predictors and other prediction techniques (where wave variables are not used at all), this technique is more robust to model uncertainties.

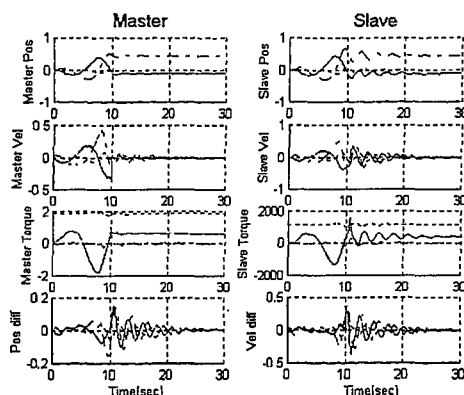


Figure 7: Joint angle, joint velocity and torque plots (no prediction).

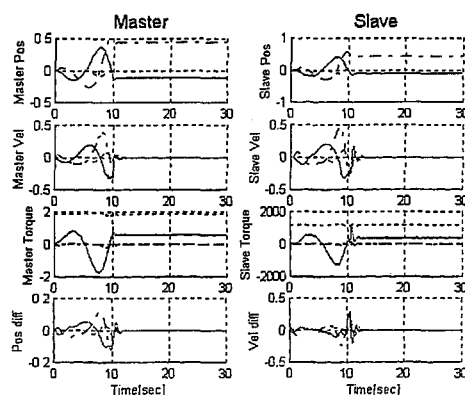


Figure 8: Joint angle, joint velocity and torque plots (with prediction).

One of the key features of this method is that passivity is explicitly enforced by the regulator. If the system model is mismatched, predicting in the wave domain while regulating the return power-flow ensures that passivity is always preserved. If prediction was done without the use of wave variables, then power-flow can not be monitored (in a straightforward manner). Another key advantage of this system is that it does not require measurement of the forces exerted by the environment onto the slave to function (although external inputs will degrade the effectiveness of the predictor).

This method is most effective for delays which are of a significant fraction (or multiple) of the smallest time constant of the system. Also the tuning parameters  $\alpha$  and  $\beta$  need some tweaking for good results. Currently work is underway to extend these results to where they can be applicable under a varying time delay, as in the case of the Internet. Physical experiments are also planned.

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